Abstract Reasoning via Logic-guided Generation

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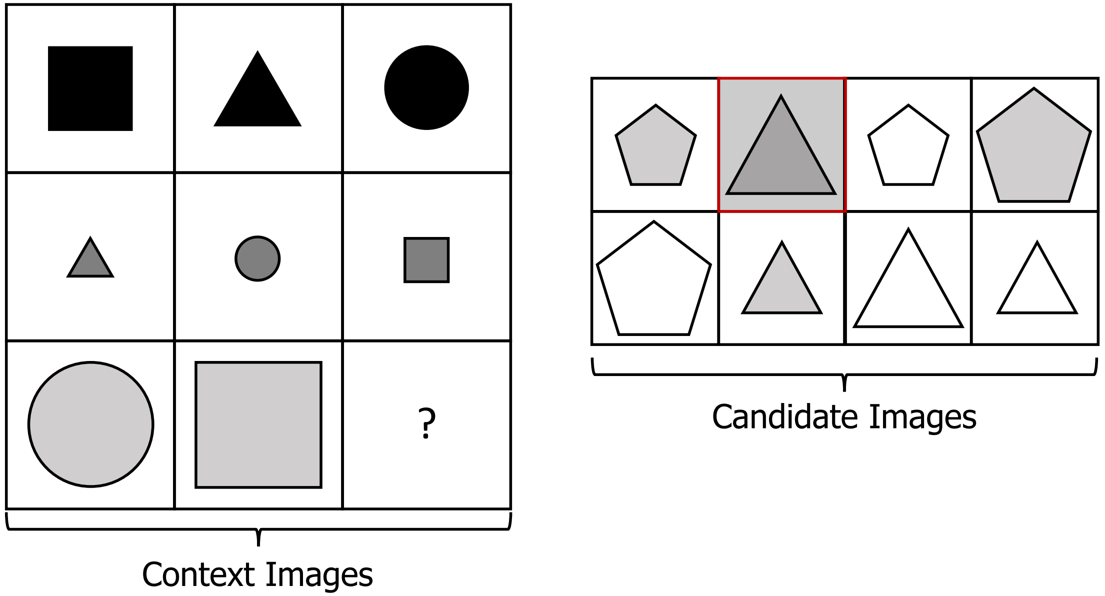
Introduction: Abstract Reasoning

Abstract reasoning aims to find out **complex patterns** based on an abstraction of observations

- One of the core characteristics of human intelligence.

Mainly evaluated by Raven's Progressive Matrices (RPM)

- Ask to identify the missing element that completes the pattern in context images
- Common format of human IQ test



DNN to Solve Abstract Reasoning Problem is Non-trivial

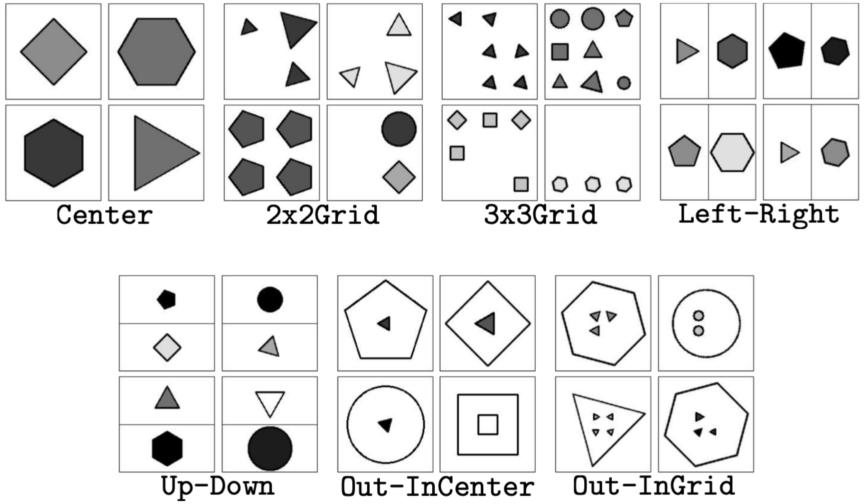
Conventional DNN architectures fail to solve the problem! [Santoro et al., 2018; Zhang et al., 2019]

- **e.g.)** ResNet [He et al., 2016], LSTM [Hochreiter et al., 1997] achieves a random accuracy

Recent approach: new framework design **motivated by a human reasoning procedure**

- Shows **improved results** on abstract reasoning benchmarks

Model	Test Accuracy(%)							
	Average	Center	2Grid	3Grid	L-R	U-D	O-IC	O-IG
LSTM	12.5	12.3	13.3	12.8	12.7	10.3	12.9	13.1
CNN+MLP	12.9	12.9	13.2	12.7	11.5	13.5	12.9	13.7
Resnet-18	14.5	20.8	12.9	14.3	13.2	13.4	13.8	12.9
WReN	17.8	23.3	18.1	17.4	16.5	15.2	16.8	17.3
LEN	28.4	42.5	21.1	19.9	27.6	28.1	32.9	27.0
CoPINet	38.6	50.4	30.9	28.5	40.0	40.8	42.7	36.9



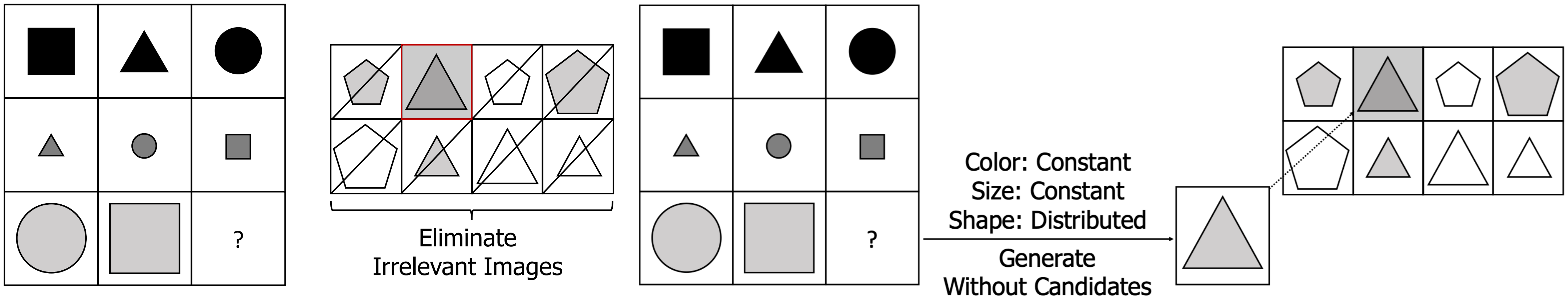
[Santoro et al., 2018] Measuring Abstract Reasoning in Neural Networks, ICML 2018
[Zhang et al., 2019] RAVEN: A Dataset for Relational and Analogical Reasoning, CVPR 2019
[He et al., 2016] Deep Residual Learning for Image Recognition, CVPR 2016
[Hochreiter et al., 1997] Long Short-term Memory, Neural Computation 1997

Two Strategies to Solve Abstract Reasoning Problem

Two different strategies to solve abstract reasoning problems. [Bendell-Fox et al., 1984; Carpenter et al., 1990]

- Discriminative strategy: eliminating irrelevant candidates – most existing DNN-based approaches
- Generative strategy: first generate without candidates, then match the answer– hardly investigated

Our contribution: design a generative framework to solve the abstract reasoning problem



[Bendell-Fox et al.,1984] Adaptive Reasoning: Componential and Eye Movement Analysis of Geometric Analogy Performance, Intelligence, 1984
[Carpenter et al., 1990] What One Intelligence Test Measures: A Theoretical Account of the Processing in the Raven Progressive Matrices Test, Psychological Review, 1990

Abstract Reasoning with Propositional Logic

Our goal: design a generative framework to solve the abstract reasoning problem

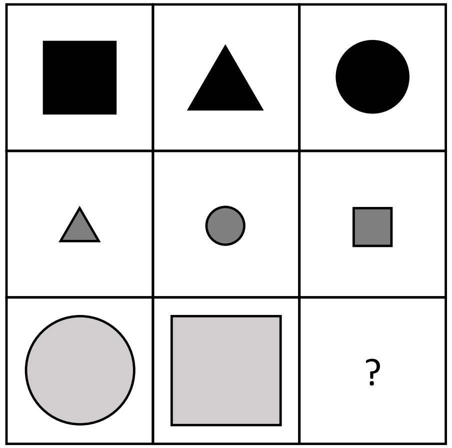
Question: what can be an effective prior for this generative framework?

Idea: reduce the problem into an optimization in propositional logic!

Key idea: most abstract reasoning problems are reduced to **MAXSAT problem**

- MAXSAT: combinatorial optimization problem in propositional logic

Unknown; only images are given

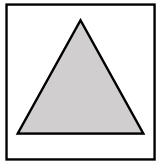


Shape: $\bigwedge_{i=1}^{3} \bigvee_{a_1,a_2,a_3\in\mathcal{A}} \bigwedge_{k=1}^{3} \bigvee_{j=1}^{3} p(i,j,a_k)$

Color: $\bigwedge_{i=1}^{3} \bigvee_{c\in\mathcal{C}} \bigwedge_{j=1}^{3} p(i,j,c)$

Size: $\bigwedge_{i=1}^{3} \bigvee_{s\in\mathcal{S}} \bigwedge_{j=1}^{3} p(i,j,s)$

$\xrightarrow{\text{Solve Logical Problem}}$



$p(3,3,\text{Triangle})$

$p(3,3,\text{Light gray})$

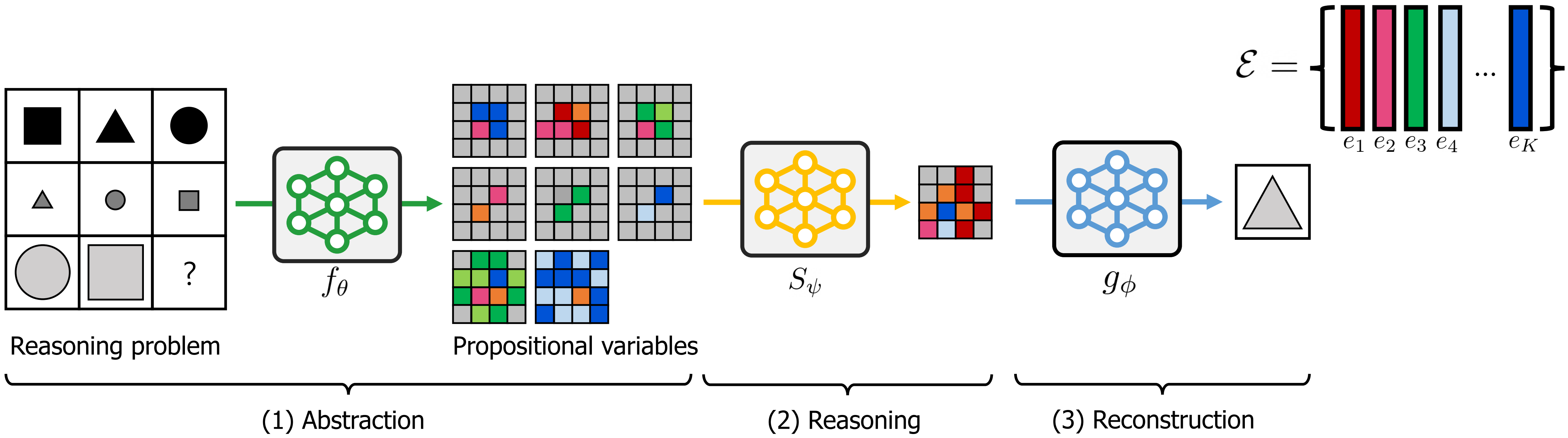
$p(3,3,\text{Large})$

LoGe: Logic-guided Generation

🤔 Can we leverage this propositional logical prior only with images?

💡 LoGe: we propose a generative framework which employs discrete embeddings from images

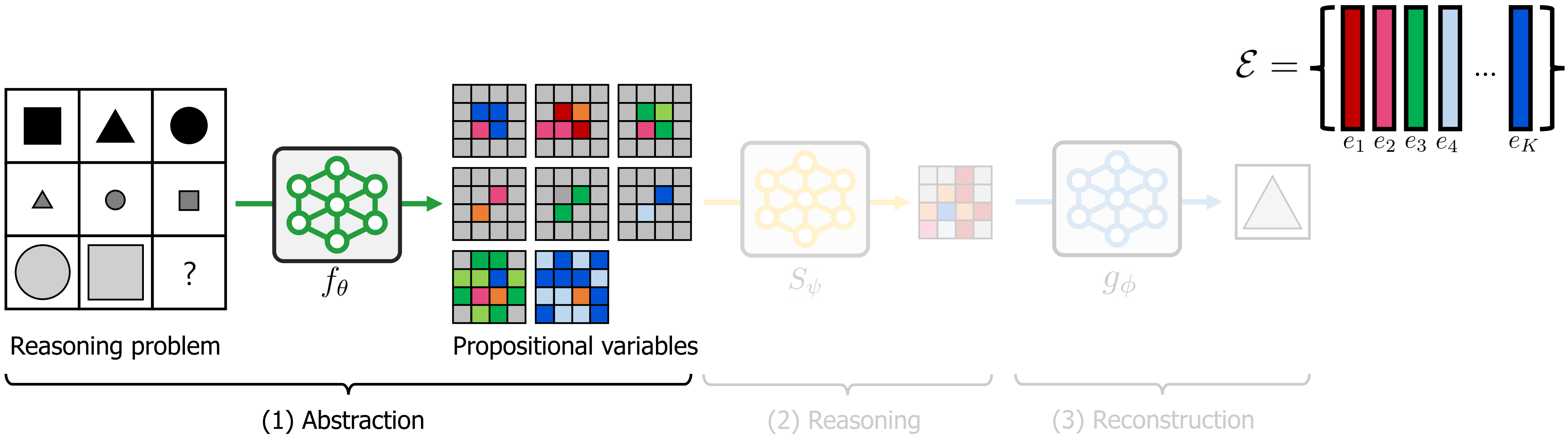
- End-to-end **three step framework**: Abstraction, Reasoning, and Reconstruction.
- **Idea:** we view discrete embeddings from images as propositional variables



LoGe: Logic-guided Generation

1. (Abstraction) LoGe embeds images into discrete embeddings

- Latent vectors from the encoder network are quantized like in VQ-VAE [Van Den Oord et al., 2017]
- We rather focus on "indices of quantized vectors"; **treat these indices as propositional variables**

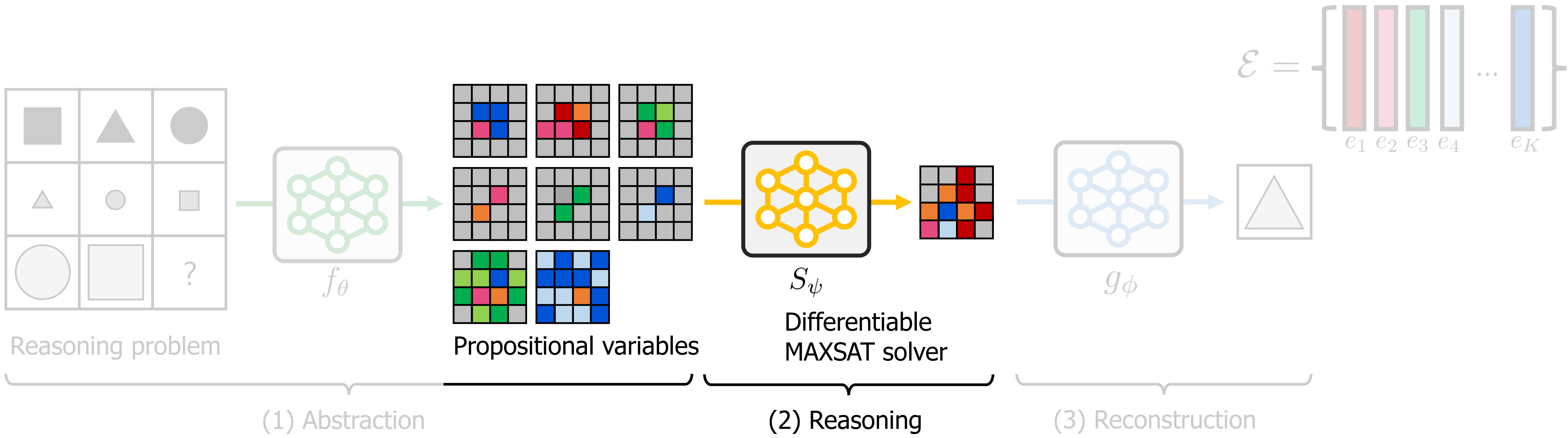


[Van Den Oord et al., 2017] Neural Discrete Representation Learning, NIPS 2017

LoGe: Logic-guided Generation

2. (Reasoning) LoGe predicts propositional variables of the answer image

- Incorporating SATNet [Wang et al., 2019], a differentiable solver layer for the MAXSAT problem
- Learns underlying propositional logical formula (rules) as layer weights from data

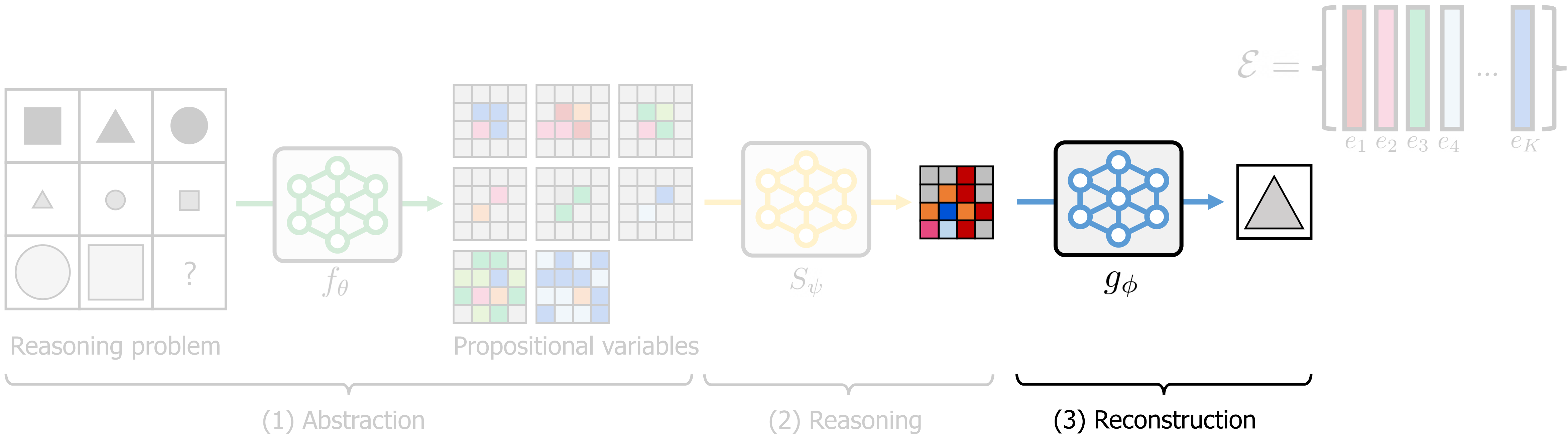


[Wang et al, 2019] SATNet: Bridging Deep Learning and Logical Reasoning Using a Differentiable Satisfiability Solver, ICML 2019

LoGe: Logic-guided Generation

3. (Reconstruction) LoGe generates the answer image based on predicted image

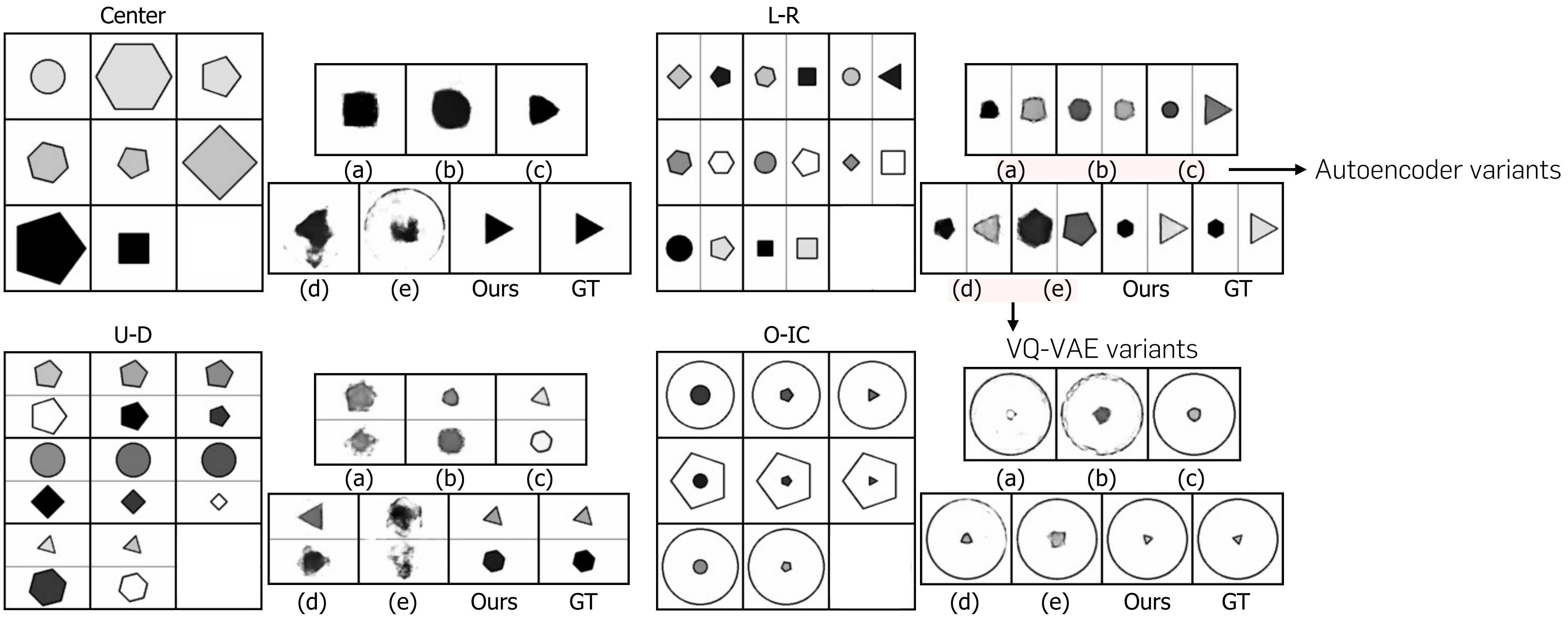
- We first convert propositional variables into quantized latent vectors from codebooks
- Image is generated from the decoder network and the converted latent representation



Experiments: LoGe Improves Reasoning Ability

LoGe shows significantly improved reasoning ability, both in qualitative and quantitative results

- **Note:** other variants without the propositional logical prior fails to generate the answer



Experiments: LoGe Improves Reasoning Ability

LoGe shows significantly improved reasoning ability, both in qualitative and quantitative results

- **Note:** generative strategy can be more effective than discriminative approach
- …even we have not accessed to wrong candidate while training!

Method	Center	U-D	L-R	O-IC
LSTM (Zhang et al., 2019a)	12.3	10.3	12.7	12.9
WReN (Santoro et al., 2018)	23.3	15.2	16.5	16.8
LEN (Zheng et al., 2019)	42.5	28.1	27.6	32.9
CoPINet (Zhang et al., 2019b)	50.4	40.8	40.0	42.7
SRAN (Hu et al., 2021)	53.4	43.1	41.4	44.0
LoGe (Ours)	**87.5**	**64.0**	**51.7**	**48.5**

Choose candidate with **smallest MSE**

Summary

In summary, we employ a propositional logic for the generative strategy in abstract reasoning

We propose a framework to leverage this knowledge only with images without symbolic labels

- Discrete embeddings from images as propositional variables
- Predict propositional variable of the answer image
- Generate the answer based on the predicted variables

Future work: effective reduction for more complicated patterns?

- e.g.) arithmetic pattern from images?

You can check out more details of our work at ariXiv: https://arxiv.org/abs/2107.10493

Thank you for your attention 😄